# RISK AVERSE APPROACHES IN OPEN-PIT PRODUCTION PLANNING UNDER ORE GRADE UNCERTAINTY: A ULTIMATE PIT STUDY

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## ABSTRACT

During early phases of open-pit mining production planning many parameters are uncertain, and since the mining operation is performed only once, any evaluations based only on *on average* outcomes neglects the very real chance of obtaining an outcome that is below average. Taking into account also that operation costs are considerable and the mining horizon usually extends over several decades it is clear that open-pit production planning is a risky endeavor. In this work we take a risk-averse approach on tackling uncertainty in the ore-grades. We consider an extended *Ultimate-Pit* problem, where extraction and processing decisions have to be taken. We apply and compare the risk-hedging performance of two approaches from optimization under uncertainty: minimize *Conditional Value-at-Risk* (CVaR) and minimization of a combination of expected value and CVaR. Additionally, we compare two decision schemes: a *static* variant, where all decisions have to be taken "now", and a *two-stage* or *recourse* variant, where we take extraction decisions now, then we see the real ore-grade, and just then processing decision is taken. Our working assumption is that we have available a large number of ore-grade scenarios. Computational results on one small size vein-type mine illustrate how minimizing average loss provides good on-average results at the cost of having high probability of obtaining undesired outcomes; and on the other hand our proposed approaches control the risk by providing solutions with a controllable probability of obtaining undesired outcomes. Results also show the great risk-hedging potential of using multi-stage decision schemes.

## INTRODUCTION

The geological uncertainty of ore grades is crucial in mine planning and it has received significant attention in the last decade. Studies have shown that incorporating the uncertainty of the ore grades into the problem could lead to final pits 15% larger in tonnage, and adding a 10% of value, see Dimitrakopoulos (2011). The most common tool to model this uncertainty is the use of conditional simulations of orebodies, see *e.g.* Dimitrakopoulos (1998) and Benndorf and

Dimitrakopoulos (2004). For instance Dimitrakopoulos *et al.* (2007) use conditional simulations to obtain candidate plans based on several orebody models, and Whittle and Bozorgebrahimi (2004) use conditional simulations to generate so-called Hybrid Pits based on set-theory. Marcotte and Caron (2012) estimate the maximum expected value of mine when extraction decisions rely on uncertain information but by the time of processing all information is known. A characteristic of these approaches is that they aim to optimize the expected profit of the operation, an eminently risk-neutral approach; however this may be inconvenient in situations where the operation is not being repeated continuously under similar conditions –contrast with operations in airline industry–, moreover when losses at stake are considerable. In view of this, and in an attempt to directly include risk control into the optimization model, Vielma *et al.* (2009) proposed a chance-constrained model that looks for solutions with low probability of obtaining low profits. The work we present here is a follow up on the latter one and builds upon its idea of using an optimization process which directly assesses the risk of feasible solutions.

In this paper we propose several risk-averse optimization methods that take into account the uncertainty in the ore-grade parameter. Our basic working assumption is that we have access to a pool of independent and identically distributed joint ore-grade vectors. The main advantage of this approach is that in this way we separate the optimization model from the geo-statistical technique that models the mineralization of the orebody. We work on the Ultimate-Pit problem, since it is a simple yet relevant aspect of life of mine planning, and consider an extended version where extraction and processing decisions have to be taken. We present and apply two risk-averse optimization approaches whose theoretical properties have been well studied, and also propose a two-stage decision scheme where after extraction occurs we can see the true mineralization of the extracted blocks. We show computational results on a small size vein-type mine and exhibit the practical strengths and weaknesses of the proposed models.

The rest of this text is organized as follows. In Section 2 we present the extended Ultimate-Pit problem on which we work. In Section 3 we introduce two decision paradigms available for the problem, and illustrate their differences by two opposite optimization models in terms of risk. In Section 4 we describe the two risk-averse optimization approaches we propose to use for the UPIT problem, and go through their most important properties in the theory of risk-averse optimization. In Section 5 we show computational results made on a small size mine and show the performance of our proposed method. Lastly, in Section 6 we state the conclusions of our work.

## THE ULTIMATE-PIT PROBLEM

The Ultimate-Pit problem (UPIT), also known as *uncapacited open-pit mine planning problem*, consists in finding the set of blocks which should be ultimately extracted, in the absence of capacity constraints, in order to maximize the value of the mine. Despite its simplicity, UPIT is a key problem for mine planning. In Caccetta and Hill (2003) they prove that the optimal multi-period capacitated open-pit mine planning problem is included in the solution of the UPIT. Moreover, a series of UPIT problems are solved to construct the *nested pits*, see Whittle (1988), which is the basis of the most common methodology to schedule open-pit mines.

We consider the following extended UPIT problem, where extraction as well as processing decisions have to be made. Let $B$ be a set of blocks, each one with an associated cost $c_b$ --a

negative cost is a profit-- for all $b \in B$. Let $P \subseteq B \times B$ be a set of precedences, that is, if $(a,b) \in P$ then in order to extract block $a$, block $b$ should also be extracted. Only if it is extracted a block can be processed, and an extracted one can be either processed or discarded. Extraction of block $b$ incurs a cost of $c_b^e$; if it is discarded it does not incur any additional cost, but if it is processed it incurs an additional cost of $c_b^p - \rho_b p_b$, where $c_b^p$ is the processing cost, $\rho_b$ is its mineral content or ore grade, and $p_b$ is the unitary profit for processing a unit of mineral in it. A mathematical formulation of the UPIT optimization problem is

$$min \quad L^{stat}(x^e, x^p, \rho) := \sum (c_b^p - \rho_b p_b) x_b^p + c_b^e x_b^e \quad s.t. \quad \left( x^e, x^p \right) \in X^{EP} \tag{1}$$

where

$$X^{EP} := \left\{ x^e, x^p \in \{0,1\}^B : x_a^e \le x_b^e \ \forall (a,b) \in P, \quad x_b^p \le x_b^e \ \forall b \in B \right\}$$

and where binary variables $x_b^e$ and $x_b^p$ indicate if block $b$ should be extracted (resp. processed) or not. Of course, $x_b^p = 1$ if and only if $x_b^e = 1$ and $c_b^p - \rho_b p_b < 0$, which allows to decide a priori if a block will be processed if extracted; with this, one can eliminate the processing variables in (1) and include all the processing decisions and costs in the extraction costs, as in Marcotte and Caron (2012). We choose however to skip this preprocessing step and build upon the formulation (1) for clarity of exposure, and also because this formulation, applied with the risk-averse models shown hereafter, is readily extendible to other mining models and constraints, *e.g.* one can include constraints on the processing decisions, include planning over several time periods, *etc*.

## UPIT UNDER ORE-GRADE UNCERTAINTY

### Uncertainty Model for the Ore Grades

A weakness of problem (1) is that it does not account for the inherent uncertainty of the estimated ore grade vector $\rho$. Indeed, the precise mineral composition in each location of the orebody is only partially known since the only information available are exploration drillings. To assess for this uncertainty we assume in this work that, first, the ore vector is actually a random vector $\tilde{\rho}$ in $\mathfrak{R}_+^B$, and second, that we can take as many independent and identically distributed (i.i.d.) samples of it. Note that this assumption allows modeling non-i.i.d. samples, and also capturing correlations between different blocks. Note also that thus we are not requiring knowledge of the actual joint distribution of the random vector $\tilde{\rho}$.

### Static vs. Two-Stage Decision Schemes and Basic Risk Models

We first note that the following formulation is completely equivalent to (1):

$$min \quad L^{rec}(x^e, \rho) := \sum c_b^e x_b^e + Q(x^e, \rho) \quad s.t. \quad x^e \in X^E \tag{2}$$

where

$$Q(x^e, \rho) := \quad min \quad \sum_{b \in B} (c_b^p - \rho_b p_b) x_b^p$$

$$s.t. \quad x_b^p \leq x_b^e \qquad \forall b \in B$$

$$x_b^p \in \{0,1\} \qquad \forall b \in B$$

and

$$X^E := \left\{ x^e \in \{0,1\}^B : x_a^e \leq x_b^e \quad \forall (a,b) \in P \right\}.$$

This formulation emphasizes the dependence of the processing decision on the extraction decision. An apparently unnecessary complication of (1), both formulations in fact differ when uncertainty is introduced. Indeed, suppose $\rho$ is now a random vector $\tilde{\rho}$ and we want, for instance, to minimize the expected loss; since we are assuming only availability of an i.i.d. sample $\rho^1, \ldots, \rho^N$, $N$ as big as we want, then we can either minimize the average loss of (1):

$$min \quad \frac{1}{N} \sum_{k=1}^{N} L^{stat}(x^e, x^p, \rho^k) \qquad s.t. \qquad (x^e, x^p) \in X^{EP}, \qquad (3)$$

or the average loss of (2):

$$min \quad \frac{1}{N} \sum_{k=1}^{N} L^{rec}(x^e, \rho^k) \qquad s.t. \qquad x^e \in X^E \qquad (4)$$

both computationally tractable formulations.

Note that (3) gives an extraction and processing plan that should be used in any scenario, and in contrast (4) gives an extraction plan to use in any scenario and a processing plan for each scenario. Essentially, (4) uses a *two-stage* decision scheme where extraction decision is taken now, then the real ore-grade is revealed, and only then processing decision needs to be taken; and (3) uses a *static* decision scheme, where all decisions need to be taken now. Using the terminology of optimization under uncertainty community, we say then that (3) is a *static* approach, and (4) is a *recourse* approach, since in the latter we have the extra recourse of "seeing" the uncertain parameter before deciding the processing decision.

We remark also that Marcotte and Caron (2012) work with the recourse scheme (4) since it is a better approach than (3) when estimating the value of mine, under the hypothesis that at later stages of the operation there will be better ore-grade estimates. Note that, when optimizing the average loss, the static approach (*i.e.* (3)) is equivalent to just plugging-in the average ore-grade in (1), so essentially Marcotte and Caron (2012) are comparing working with the average ore-grade and working with profit scenarios. In contrast, we seek to compare the static decision scheme with the recourse one under several risk-averse optimization models.

Note that minimization of average losses is essentially a risk-neutral approach, since it ignores the distribution of below- and above-average losses. Although by design this approach is the one that will provide the best on-average outputs, it can return solutions with high probability of obtaining big losses. On the contrary, an extreme risk-averse approach would be to minimize de worst possible loss. In this case we are sure that the obtained solution will do well –better than any other solution, by design– even in the worst possible scenario. A computationally tractable formulation, for the static and the recourse versions, is easily obtained by minimizing the maximum-loss scenario when considering losses over a finite sample of ore-grades. We consider then minimization of expected- and worst-loss as the two opposite benchmarks when evaluating the performance of the risk-averse approaches presented in the following section.

## RISK-AVERSE MODELS FOR UPIT UNDER UNCERTAINTY

We now introduce the two risk-averse optimization models that we compare in our work: optimization of Conditional Value-at-Risk model and $\varepsilon$-Modulated Convex-Hull model. A closely related approach to the first model is the optimization of Value-at-Risk model, in which one optimizes the $\varepsilon$-quantile of the worst losses, as in Vielma *et al.* (2009). However, in Lagos *et al.* (2011) we showed that this model returned plans with good Value-at-Risk but at the expense of having higher chance of exhibiting bad losses. This inadequate manage of risk lastly deems the model inappropriate for the use in large scale open-pit mining planning problems, and such is the reason we do not include it in this study.

All the models in this section can be applied to both the static and the recourse decision schemes, however for the sake of space economy we explain them only for the static version. To obtain the recourse versions in all models it is sufficient to replace $L^{stat}(x^e, x^p, \rho)$ by $L^{rec}(x^e, \rho^k)$, and $(x^e, x^p) \in X^{EP}$ by $x^e \in X^E$. All strategies and properties shown here will hold for the recourse variant as well: we rely on the SAA approximation too for computational tractability, we have the same risk-hedging properties, convergence of optimal value and solution set, *etc*.

### CVaR Model

Given a risk level $\varepsilon \in (0,1]$ (lower $\varepsilon$ means lower risk), and given a real random variable $\tilde{l}$ representing losses, the Conditional Value-at-Risk (*CVaR*) at level $\varepsilon$ is roughly speaking the mean of the $\varepsilon$ portion of the worst losses. In Rockafellar and Uryasev (2000) it is shown that CVaR can be defined, consistently with the previous notion, in the following way:

$$CVaR_\varepsilon(\tilde{l}) := \min_{z \in \Re} \left\{ z + \frac{1}{\varepsilon} E[\tilde{l} - z]^+ \right\}$$

where $[a]^+ = a$ if $a > 0$ and $0$ otherwise. Theoretically, CVaR is a good measure of the risk of the random loss $\tilde{l}$ since it is a *coherent risk measure* in the sense of Artzner *et al.* (1999). Moreover, it is the fundamental *distortion risk measure*, see *e.g.* Bertsimas and Brown (2009).

The first model we consider then is the *minimization of CVaR*, where given a risk level

$\varepsilon \in (0,1]$ we choose the plan $(x^e, x^p) \in X^{EP}$ that minimizes $CVaR_\varepsilon(L^{stat}(x^e, x^p, \tilde{\rho}))$. From Rockafellar and Uryasev (2000), this optimization problem can be formulated as

$$min \qquad z + \frac{1}{\varepsilon} E[L^{stat}(x^e, x^p, \tilde{\rho}) - z]^+ \qquad s.t. \qquad z \in \Re, \quad (x^e, x^p) \in X^{EP}. \qquad (5)$$

This problem, as it is, is intractable since the expected value in the objective function requires multidimensional integration and over a distribution that is unknown. Therefore we use the so-called *Sample Average Approximation* (SAA), which consists of approximating the expected value by the sample average of a finite sample of the uncertain parameter. That is, we take an i.i.d. sample $\rho^1, \ldots, \rho^N$ of the grades vector $\tilde{\rho}$ and solve instead

$$min \qquad z + \frac{1}{\varepsilon N} \sum_{k=1}^{N} [L^{stat}(x^e, x^p, \rho^k) - z]^+ \quad s.t. \qquad z \in \Re, \quad (x^e, x^p) \in X^{EP}. \qquad (6)$$

This latter problem is easily linearized by including one linear variable per each scenario, obtaining thus a tractable MILP program –which applies for both the static and the recourse versions.

We call (5) the *real CVaR problem* and (6) the *SAA CVaR problem*. SAA approximations have been widely studied, see *e.g.* Linderoth *et al.* (2006), and under mild assumptions (see Lagos, 2011, and §5.1.1 Shapiro *et al.*, 2009) it holds that the SAA CVaR problem is consistent with the real CVaR problem, in the sense that as the number *N* of samples increases the optimal value and set of optimal solutions of (6) converges to the respective counterparts of (5).

## MCH-$\varepsilon$ Model

The second model we consider is the so-called *ε-Modulated Convex-Hull* (MCH-ε) model. This approach is a modification of the *MCH model* proposed in Lagos *et al.* (2011), where a modulated convex-hull of an i.i.d. sample was taken as the uncertainty set in a robust-type formulation. Although the nice theoretical properties of this latter approach, the advantage of MCH-ε model is that its underlying risk measure does not depend on the sample size *N*, and is easily extendible to a two-stage decision scheme, as will be seen in Section 3.

Consider a risk level $\varepsilon \in (0,1]$. The MCH-ε model consists on choosing the plan that solves

$$min \qquad \varepsilon E\left[L^{stat}(x^e, x^p, \tilde{\rho})\right] + (1-\varepsilon) CVaR_\varepsilon\left(L^{stat}(x^e, x^p, \tilde{\rho})\right) \qquad s.t. \qquad (x^e, x^p) \in X^{EP} \qquad (7)$$

Theoretically this model is attractive since we are minimizing a well-studied distortion risk measure of the losses that could be a better estimator of the CVaR-ε, see Lagos *et al.* (2012). Now, as in the CVaR model, this model, as it is, is intractable since it requires high dimensional integration and knowledge of the precise distribution of the ore grades vector $\tilde{\rho}$. Hence we proceed as before and use the SAA approximation to solve an approximated model: we take an i.i.d. sample $\rho^1, \ldots, \rho^N$ of $\tilde{\rho}$ and approximate $E\left[L^{stat}(x^e, x^p, \tilde{\rho})\right]$ and $CVaR_\varepsilon\left(L^{stat}(x^e, x^p, \tilde{\rho})\right)$ with the in-sample mean, to obtain the problem

$$\min \quad \varepsilon \frac{1}{N} \sum_{k=1}^{N} L^{stat}(x^e, x^p, \rho^k) + (1-\varepsilon)\left[ z + \frac{1}{\varepsilon N} \sum_{k=1}^{N} [L^{stat}(x^e, x^p, \rho^k) - z]^+ \right]$$

(8)

$$s.t. \quad z \in \Re, \quad (x^e, x^p) \in X^{EP}.$$

This problem is easily linearized by adding one linear variable per scenario, obtaining a tractable MILP program. As in the CVaR model, convergence, as the number of samples $N$ increases, of the optimal value and set of the problem (8) to the respective counterparts of the "true" problem (7) is *a.s.* assured under mild assumptions, see Lagos (2011) and §5.1.1 Shapiro *et al.* (2009).

## COMPUTATIONAL EXPERIMENTS

We apply these models on a small size vein-type mine, for which an i.i.d. sample of 20,000 ore-grades vectors are available using Emery and Lantuéjoul (2006). The mine consists of 6,630 $10 \times 10 \times 10m^3$ blocks from a real orebody, and when adding a crown of zero mineral content on the sides we obtain a total of approx. 19,700 blocks and approx. 86,450 extraction precedences. The extraction and processing costs per block are respectively 1 and 5 monetary units, and each mineral unit processed produces a benefit of 25 monetary units. Each experiment consists of the following: choose one of the four models (CVaR static, MCH-$\varepsilon$ static, CVaR recourse or MCH-$\varepsilon$ recourse), choose a risk level $\varepsilon \in (0,1]$, choose a sample size $N$ and choose at random $N$ ore-grades vectors out of the 20,000 pool available; with this solve the SAA approximation of the model, thus obtaining a candidate production plan. All MILP programs are solved with the solver CPLEX v. 12. We also consider the models of, given an ore-grade sample, choose the plan that minimizes the expected in-sample loss, and choose the plan that minimizes the worst in-sample loss. Additionally, for every obtained plan we calculate the loss in each of the $N$ scenarios used for solving the model –we call them *in-sample* losses–, and also calculate the loss in a different ore-grade sample consisting of 10,000 ore-grade scenarios out of the 20,000-$N$ samples left –this we call the *out-of-sample* losses. The idea is that the in-sample losses provide insight of "what the model thinks it is doing", and the out-of-sample losses of "what the model is actually doing".
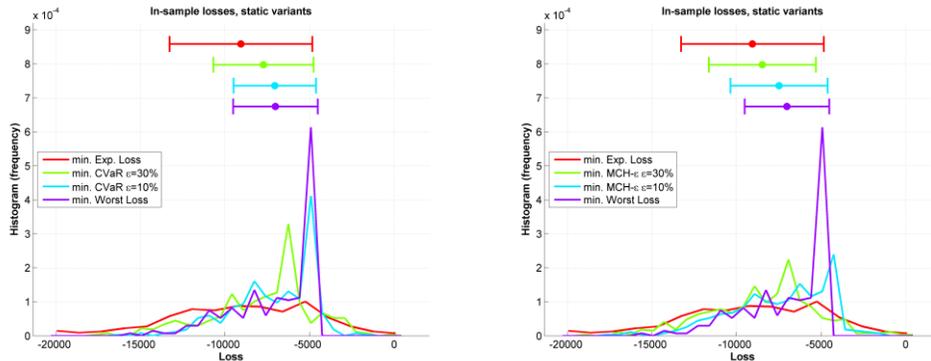
**Figure 1** – In-sample results for CVaR (static) and MCH-$\varepsilon$ (static) models.

In Figure 1 we show histograms of the in-sample losses for the static approaches and for several risk levels $\varepsilon$; and above each histogram it is plotted a bar that shows the mean and standard deviation of the losses of each plan. Since we do not know the real value of the ore-grade vector, histograms provide an idea of the distribution of the losses of the plan obtained with each model. This is a fair evaluation method since, to the best knowledge of the authors, there is no robust method of choosing which scenario is the "real" one. Results in Figure 1 are obtained taking the same sample, which is of size $N=400$ scenarios. These results show that indeed in all models the risk level $\varepsilon$ controls how much chance we have of obtaining a bad loss, and also that decreasing $\varepsilon$ we transit from the performance of minimizing expected loss to minimizing the worst loss –the riskier approach to the most conservative one. Most importantly, it is clear that minimization of expected loss indeed attains the best average loss, however it achieves this by allocating the highest probability on obtaining good as well as bad losses, confirming that such an approach provides risky solutions. On the other hand the risk-averse models put less probability on bad losses, at the cost of increasing the expected loss though; however the trade-off between this two effects is controllable with the risk level $\varepsilon$.
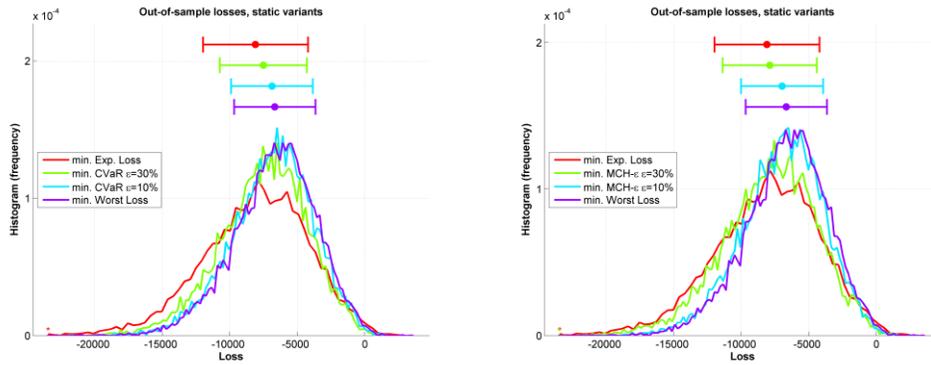


**Figure 2** – Out-of-sample results for CVaR (static) and MCH-$\varepsilon$ (static) models.

In Figure 2 we show the out-of-sample performance of the same plans obtained for Figure 1. That is, for the eight plans whose histograms are shown in Figure 1 we compute their out-of-sample losses and plot their histograms in Figure 2. Note that when evaluating out-of-sample the plans there is not much difference between performances of different models. This loss of resolution is observed for the MCH-$\varepsilon$ model too, and could imply that the in-sample size, $N=400$, is not a representative sample of the distribution of $\tilde{\rho}$; ultimately an undesirable outcome.
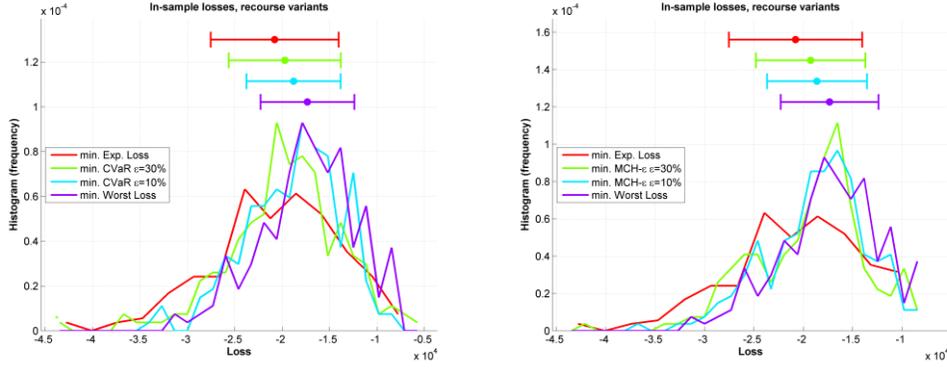
**Figure 3** – In-sample histograms for CVaR (recourse) and MCH-$\varepsilon$ (recourse) models.

In Figure 3 we show the in-sample losses we obtain with CVaR recourse and MCH-$\varepsilon$ recourse models, where we solve the SAA approximations with $N=200$ samples of the grades vector $\rho$.

Note that with recourse models the outcome between models and risk levels is very similar. But remarkably we only obtain negatives losses, *i.e.* profits, which is a dramatic improvement over the non-recourse models. We recall though that recourse variants use a different decision scheme than static variants, so both approaches are not directly comparable. Nonetheless, this results show that there is a great risk hedging potential in implementing multi-stage decision schemes in mining production planning. We also mention that we obtain the same loss of resolution, as in non-recourse models, when passing from in- to out-of-sample performance.

**Table 1** – 95% confidence intervals for optimal value of respective "real" problems.

|  | $\varepsilon = 5\%$ | $\varepsilon = 10\%$ | $\varepsilon = 30\%$ | $\varepsilon = 50\%$ |
|---|---|---|---|---|
| CVaR stat. *N=400* | [-3,835, -1,200] | [-4,012, -1,934] | [-4,923, -3,878] | [-5,843, -5,110] |
| MCH-$\varepsilon$ stat. *N=400* | [-3,988, -1,551] | [-4,339, -2,505] | [-5,889, -5,135] | [-7,143, -6,575] |
| CVaR rec. *N=200* | --- | [-11,780, -10,623] | [-14,031, -13,324] | [-15,744, -15,172] |
| MCH-$\varepsilon$ rec. *N=200* | --- | [-12,548, -11,552] | [-15,869, -15,303] | [-18,065, -17,569] |

We also implement a repetitions and screening procedure as shown in §5.6.1 Shapiro *et al.* (2009), where for a given model and risk level $\varepsilon$ we take $M=30$ i.i.d. samples of the same size $N$, and obtain an optimal plan in each of the $M$ samples. This ultimately allows to obtain a 95% confidence interval for optimal value of the "real" model. Indeed, recall that for CVaR and MCH-$\varepsilon$, static and recourse versions, we first proposed a "real" problem (*e.g.* (5)) related to the "real" distribution of $\tilde{\rho}$, and due to its intractability we resorted to solve its SAA

approximation by taking i.i.d. samples (*e.g.* (6)). This confidence interval then provides an estimation of the optimal value for the "true" problem, and note that in particular for CVaR models it provides an estimation of the "true" minimal CVaR possible by a feasible plan. We refer the reader to §3.3 Lagos (2011) for further details on this procedure. In Table 1 we show the aforementioned 95% confidence intervals for CVaR and MCH-$\varepsilon$ models, static and recourse versions, for several risk levels $\varepsilon$. Recourse models with $\varepsilon=5\%$ could not be solved due to either shortage of memory or no near-optimal solution was found in less than 24 execution hours, so those fields in Table 1 are left blank. In this regard, the preprocessing step mentioned in the second section could have made this problem easily solvable, however in this work we focused solely in evaluating the risk hedging performances of the proposed models. We note that all interval widths are considerable and as the risk level $\varepsilon$ increases the width of the interval decreases. A heuristic argument for this phenomenon is that with lower risk levels $\varepsilon$ we are focusing on a smaller fraction of the worst losses, which are the most difficult to observe, so then we need a larger sample size.

**Table 2** – Expected profit estimator for optimal plan of each model.

|  | Minimize Worst Loss | $\varepsilon = 10\%$ | $\varepsilon = 30\%$ | $\varepsilon = 50\%$ | Minimize Exp. Loss |
|---|---|---|---|---|---|
| CVaR stat. *N=400* | 6,670 | 6,878 | 7,519 | 7,914 | 8,106 |
| MCH-$\varepsilon$ stat. *N=400* | 6,670 | 6,979 | 7,898 | 8,069 | 8,106 |
| CVaR rec. *N=200* | 17,621 | 18,754 | 19,791 | 19,894 | 20,123 |
| MCH-$\varepsilon$ rec. *N=200* | 17,621 | 19,254 | 19,919 | 20,064 | 20,123 |

Finally, in Table 2 we show, for each model and several risk levels, an expected profit estimator obtained with the out-of-sample profits (*i.e.* the average of 10,000 profits). Additionally we include the expected profit of the plans obtained with the models of minimizing worst loss and minimizing expected loss. Static models are solved using *N=400* samples and recourse ones with *N=200*. Clearly as the risk level increases the expected profit increases. And again, the increment in the profit due to using the recourse decision-scheme is dramatic, so much that even the most risky non-recourse model (min. exp. loss) attains roughly half of the expected profit of the most conservative recourse approach (min. worst loss).

## CONCLUSIONS

- We propose two risk-averse approaches for the ultimate-pit problem under ore-grade uncertainty, and also model two possible decision schemes. We resort to the SAA approximation to obtain tractable MILP programs. These models and their approximations have good theoretical properties in the risk-averse optimization framework.

- Computational results show that the classic approach of minimizing expected losses indeed attain the best on-average results, however such solutions exhibit the highest probability –between the models compared– of obtaining undesirable outcomes. On the contrary, the proposed risk-averse models control how much probability the decision maker is willing to accept of getting undesired losses.

- Computational results also indicate that in the proposed models the risk level $\varepsilon$ gives a fine control of the riskiness –or uncertainty of the final outcome– of the obtained plan. However more certainty is achieved at the expense of obtaining on average higher losses.

- Results also suggest that, even for a small size mine, a very big sample of the ore grades is needed to capture the distribution of the joint ore grade vector. This is ultimately a weakness of the chosen modeling, since as the sample size grows the problems require more computational resources to be solved.

- Although the variants with recourse present the same theoretical properties and practical weaknesses as the static variants –*e.g.* need for a much bigger sample size to obtain consistency between in- and out-of-sample performance–, the two-stage paradigm shows a dramatic improvement over the static one, since it only attains negative losses. And even though there is not much difference between the performance of the evaluated recourse variants, even using minimization of expected loss provides a good protection to uncertainty in the ore-grade. This great potential suggests exploring the feasibility of implementing in practice sequential decision plans where relevant decisions are delayed until uncertain parameters are better estimated.

## ACKNOWLEDGEMENTS

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